November-2019

BCA, Sem.-I

CC-104 : Fundamental Mathematical Concepts (FMC)
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (i) Let U= {–2, –1, 0, 1, 2, 3, 4}, A = {0, 1, 2, 3, 4} and B = {–2, –1, 0, 1, 2, 3}. Also consider f:A → B, $f(x) = x - 1$, and g:B → A, $g(x) = |x|$. Using above information answer the following questions. 7

(1) A ∪ B' (' is the symbol for complement)

(2) A – B

(3) R_f (range of f)

(4) R_g (range of g) .

(5) $R_f \cup R_f$

(6) $R_f \cap R_f$

(7) $(R_f \cup R_f)'$

(ii) If $p^x = q^y = pq$ then verify that $\frac{1}{x} + \frac{1}{y} = 1$, where p, q ∈ $R^+ - \{1\}$. 7

OR

(i) Show that the function f:Z → Z, $f(x) = 2x + 3$, is one-one and onto. Also find the inverse f^{-1} of f and show that $fof^{-1} = f^{-1}of$.

(ii) Let A = {1, 2, 3, 4} and B = (2, 4, 6, 8}. Then verify the equalities, $(A \cup B)' = A' \cup B'$ and A – B = A – (A ∩ B).

(B) Do as Directed : (Any **Four**) 4

(i) If $A = \{x \mid x \in Z, x^4 - 16 = 0\}$, then set A is equals to

(a) {–2, 2} (b) {–2, 0, 2}

(c) {16} (d) None of these

(ii) Which of the following is true ?

(a) $A \subset B$ (b) $A = B$

(c) $A \cap B = \varnothing$ (d) None of these

(iii) Let $A = [1, 3]$ and $B = [2, 5]$. Then $A \cap B =$ _____

(a) (2, 3) (b) [2, 3]

(c) (2, 3] (d) [2,3)

(iv) Graph of the identity function $I : R \to R$, $f(x) = x$, passes through

(a) (1, 2) (b) (0, 0)

(c) (2, 3) (d) None of these

(v) Let $f : A \to B$ and $g : C \to D$. Then fog exists if,

(a) $R_f \subseteq D_g$ (b) $R_g \subseteq D_f$

(c) $R_f \subseteq R_g$ (d) None of these

(vi) Range of the function $f : A \to B$, is

(a) Subset of A (b) Subset of B

(c) Proper subset of A (d) Proper subset of B

2. (A) (i) (a) Let A be a square matrix given below. Prove that A can be written as a sum of symmetric and skew-symmetric matrices. 4

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) Define Rank. Also find the rank of the above matrix 3

(ii) (a) Solve the following system using Cramer's Rule. 4

$$2x - y + z = 2$$
$$3x - y - 2z = 2$$
$$x + y + z = -3$$

(b) Verify that $A(\text{adj } A) = |A| I_3$ for the following matrix.. 3

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$$

OR

(A) (i) (a) Solve the following system using inversion Method : 4

$$x + y + z = 1$$
$$2x - y + z = 1$$
$$3x - y + z = 2$$

(b) Verify that $AA^{-1} = I = A^{-1} A$ for the following matrix. 3

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

(ii) (a) If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & 0 \\ 3 & 2 & -1 \end{bmatrix}$ then show that $A^3 = O$. 4

(b) If $A = \begin{bmatrix} 1 & -2 & 2 \\ 3 & 2 & -1 \\ 3 & -2 & 1 \end{bmatrix}$ then verify that $\text{Rank}(A+A^T) = \text{Rank}(A) + \text{Rank}(A^T)$. 3

(B) Do as Directed : (Any Four) 4

(i) A matrix $M = \begin{bmatrix} 2 & x \\ 3 & 6 \end{bmatrix}$ is invertible if $x \neq$ _____

(a) 2 (b) 3

(c) 4 (d) None of these

(ii) Rank of Identity Matrix of order 5 is

(a) 2 (b) 4

(c) 6 (d) None of these

(iii) $\begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} =$ _____.

(a) $\tan\theta$ (b) $\sec\theta$

(c) $\cot\theta$ (d) None of these

(iv) The solution set of $x = 0$ and $y = 0$ is _____.

(a) $\{(1, 0)\}$ (b) $\{(0, 1)\}$

(c) $\{(1, 0), (0, 1)\}$ (d) None of these

(v) Every diagonal matrix is a scalar matrix. (True /False)

(vi) If rank of two matrices are same then that two matrices are equal. (True / False)

(A) (i) Find the point which is equidistant from (1, 2), (3, 6) and (5, 2). Also find the area of a triangle made by these three points. 7

(ii) Find the equation of a line passing through (–1, 2) and making angle 45° with line $2x + 5y + 4 = 0$. 7

OR

(i) Find the equation of a line passing through the intersection of the lines $x - 2y - 2 = 0$ and $2x - 5y + 1 = 0$ and 7

(1) having Slope –1/2.

(2) Is perpendicular to $3x - 2y + 11 = 0$.

(3) Is parallel to $2x - 5y + 13 = 0$.h

(4) having x-intercept 2.

(ii) Find the area of a triangle formed by lines given below : 7

$7x + y - 11 = 0$, $x + 3y + 7 = 0$, $3x - y + 1 = 0$.

(B) Do as Directed : (Any **three**)

(i) Two lines are parallel if _____.

(a) their slopes are same (b) their y intercepts are different

(c) Angle between them is of 45° (d) None of these

(ii) Equation of a line passing through (0, 0) and having slope α is,

(a) $y = \alpha x$ (b) $x = \alpha y$

(c) $y = \alpha x + c$ (d) None of these

(iii) Area of a triangle made by three points (0, 0), (1, 0) and (0, 1) is _____.

(a) 0 (b) 1

(c) 2 (d) None of these

(iv) Equation of a line perpendicular to $2x - 4y + 5 = 0$ is _____.

(a) $2x + 4y + 5 = 0$ (b) $2x - 4y + 10 = 0$

(c) $2x - 4y + 15 = 0$ (d) None of these

(v) Three points (–2, –2), (1, 1) and (1, 2) are collinear points. (True / False)

4. (A) (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $y = x^2 + 2x + 3$. Also check the continuity of y at $x = 2$. 7

(ii) Let $y = \sin(2x + 3)$. Find $\frac{dy}{dx}$ and $\int y\,dx$. 7

OR

(i) Find $\frac{dy}{dx}$ and also evaluate limit as x tends to 3 for $y = \frac{x^3 - 27}{x - 3}$ 7

(ii) Evaluate : $\int_{-1}^{1}\left(x^2 + \frac{1}{3x+7}\right)dx$ 7

(B) Do as Directed : (Any **three**) 3

(i) The derivative of a constant function is zero. (True / False)

(ii) The derivative of a straight line $ax + b$ is a. (True / False)

(iii) Second derivative of $ax^2 + bx + c$ is constant. (True / False)

(iv) Definite integration gives the area under the curve bounded by x-axis and its given limits. (True / False)

(v) Integration of a constant function is a straight line. (True/False)

Seat No. : _____

ND-108

November-2019

BCA, Sem.-I

CC-104 : Basics of Mathematics
(Old)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (i) If A = {1, 2, 3}, B = {2, 3, 4, 5} and C = {2, 4, 6, 8} then verify that 7

(a) $A \cup B = (A - B) \cup B$

(b) $A - (A - B) = A \cap B$

(ii) If set A contains 71 elements, Set B contains 29 elements and A and B contains 20 elements. Then find n(A U B), n(A – B) and n(B – A) 7

OR

(i) If $f(x) = x(x - 1)(2x - 1)$ then find $f(x + 1) - f(x + 2)$. 7

(ii) A pen drive making company finds that the production cost of each Pen-drive is ₹ 30 & fixed cost is ₹ 1,800. If each pen-drive can be sold for ₹ 50 determine 7

(a) cost function (b) the revenue function (c) the B.E.P.

(B) Attempt any **Four** : 4

(1) If we are dealing with the set of all computer programmers in the world, then which of the following can be a Universal set ?

(a) Set of all men in the world (b) Set of all women in the world

(c) Set of all people in the world (d) Set of all Indians in the world

(2) If a function f : A → B is such that Range f = B then f is a/an ?

(a) Into function (b) Onto function

(c) Injective function (d) None of these

(3) Give an example of Disjoint set.

(4) If n(A) =5 and n(B) = 3, What will be the value of $n(A \times B)$?

(5) In a group of 20 children, 8 drink tea but not coffee and 13 like tea. The number of children drinking coffee but not tea is _____.

(6) If A is singleton set then its power set is empty. [True / False]

2. (A) (i) If $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & -5 & 5 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$ compute A^2B^2. 7

(ii) Find A^{-1} for matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$. 7

OR

(i) Solve the system using Cramer's Rule :

$6x + y - 3z = 5$; $2x + y + 4z = 8$; $x + 3y - 2z = 5$

(ii) For the given matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ find

(a) Determinant of a matrix A

(b) Rank of matrix A.

(c) Express matrix A as sum of symmetric and skew symmetric matrix.

(B) Attempt any **Four** : 4

(1) What do you mean by singular matrix ?

(2) What is the necessary condition for multiplying two matrix ?

(3) Give an example of matrices A and B such that $A \neq 0$, $B \neq 0$ but $AB = O$.

(4) All diagonal entries of a Skew-symmetric matrix is zero. [True/False]

(5) Inverse of an identity matrix is itself. [True/False]

(6) Rank of a matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is 2.[True/False]

3. (A) (i) Attempt the following : 7

(a) Find the ratio in which the line joining points (2, 2) and (7, 8) is divided by x-axis. Also find that Point.

(b) If the distance between the points (a, –5) and (2, a) is 10 units, find the value of a.

(ii) Attempt the following : 7

(a) Show that the points (2, –2), (8, 4), (5, 7) and (–1, 1) form a rectangle.

(b) A Line passes through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x + 3y = 4$ and it makes intercepts on the axes equal in magnitude but opposite in signs. Find its equation.

OR

(i) Attempt the following :

(a) Find the area of triangle formed by three points (4, 4), (3, –2) and (3, –16).

(b) Find the equation of a line passing through the origin and parallel to the line $3x - 2y + 1 = 0$.

(ii) Attempt the following :

(a) Find the equation of line passing through the point (2, –3) and perpendicular to the line $3x + 4y - 1 = 0$.

(b) If (2, –2), (8, 4) and (5, 7) are the vertices of a triangle, find mid-points of each side of triangle.

(B) Attempt any **Three** : 3

(1) When the slope of a line is undefined ?

(2) Find out equation of a line passing through (2, –1) and inclined at 45° to the x- axis.

(3) When we talk about an angle between two lines we always mean ____

(a) acute angle (b) obtuse angle (c) right angle

(4) X-intercept of $4x - y + 2 = 0$ is

(a) 2 (b) –2 (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(5) Two lines $x + y + 1 = 0$ and $x - y + 1 = 0$ are perpendicular. [True/ False]

4. (A) (i) Attempt the following : 7

(a) Find $\frac{dy}{dx}$ when $y = \log(\sin\sqrt{x^2 + 1})$

(b) Evaluate : $\int x^2 (x^3 + 2)^{5/3} dx$

(ii) Attempt the following : 7

(a) Find $\frac{dy}{dx}$ when $y = x^{3/2} 4^x$

(b) Evaluate : $\int 3x + 4x^5 - 5x^2 dx$

OR

(i) Attempt the following :

(a) $\lim_{x \to 7} \frac{x^2 + 2x - 63}{x^2 - 10x + 21}$

(b) $\lim_{n \to \infty} \frac{(an + h)(cn + h)(cn + f)}{n^3}$

(ii) Attempt the following :

(a) $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$

(b) Check the continuity at $x = a$.

$$f(x) = \frac{x^2}{a} - a \, , \, 0 < x < a$$

$$= 0 \quad , x = a$$

$$= a - \frac{x^3}{a^2}, x > a$$

(B) Attempt any **Three** : 3

(1) $\lim_{x \to 0} \frac{\tan x}{x} =$ _____

(a) 0 (b) 1 (c) Π (d) ∞

(2) Find $\frac{dy}{dx}$ of $y = \frac{x^4}{4} + 2x^2$.

(3) $\int \frac{2x + 1}{2x} dx =$ _____

(a) $x + \frac{1}{2} \text{in} \, | x | + c$ (b) $1 + \frac{1}{2} x^{-1} + c$

(c) $x + 2 \ln | x | + c$ (d) $x + \ln | 2x | + c$

(4) What is the second ordered derivative of $f(x) = 4x^3 - 11x^2 - 14x + 19$?

(5) $\lim_{x \to 2} \frac{x^2 - 4}{x^2 + 4} =$ _____

(a) 1 (b) 0 (c) $(-\frac{1}{2})$ (d) (-1)
