Seat No. : 533

MD-107

May-2017

B.C.A., Sem.-II

CC-111 : Mathematical Foundation of Computer Science

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Figures to the right indicate marks.

1. (A) Define Abelian group. If '$*$' is defined on the set of integers Z as $a * b = a + b + 2$, prove that $(Z, *)$ is an Abelian group. **8**

OR

Define cyclic group. Find generator of $(Z_8, +_8)$.

(B) Define order of an element in a group. Let $G = \{1, -1, i, -i\}$, where i is the square root of (-1), be a multiplicative group. Find order of every element. **6**

OR

For any $a, b, c \in G$, prove that

(i) $a * b = a * c \Rightarrow b = c$ (Left Cancellation Law)

(ii) $b * a = c * a \Rightarrow b = c$ (Right Cancellation Law)

2. (A) Define the Partition. Let $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 2), (1, 1), (2, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5)\}$ be equivalence relation on A. Determine the partition corresponding to R^{-1}, if it is an equivalence relation. **8**

OR

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. Let R and S be two relations from A to B defined by $R = \{(1, 1), (2, 2), (3, 3)\}$ and $S = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$. Find

(i) $R \cup S$ (v) Dom (R^{-1})

(ii) $R \cap S$ (vi) $(R \cup S)^{-1}$

(iii) R^{-1} (vii) $R^{-1} \cap S^{-1}$

(iv) S^{-1} (viii) Range $(R \cap S)$

(B) Define Chain. Draw the Hasse diagrams of the following sets under the partial order relation "divisibility". Also investigate for chain. **6**

(i) $\{1, 2, 4, 8\}$

(ii) $\{2, 5, 20\}$

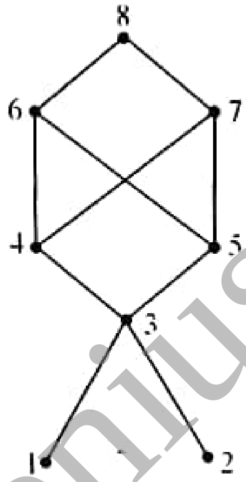
(iii) $\{1, 2, 5, 10, 20\}$

OR

Consider the POSET $P = \{1, 2, 3, 4, 5, 6, 7, 8\}$ under the partial order whose Hasse Diagram is as shown below. Consider the subsets $A = \{1, 2\}$ and $B = \{3, 4, 5\}$ of P. Find

(i) all the lower and upper bounds of A and B.

(ii) GLB(A), LUB(A), GLB(B), LUB(B)



3. (A) Express the following Boolean expression in a sum of product canonical form : **8**

(i) $x_1 * (x'_2 * x_3)'$

(ii) $x_3 * (x'_1 \oplus x_2) \oplus x'_2$

OR

In any Boolean algebra, prove that

$(a + b)(a' + c) = ac + a'b + bc = ac + a'b$

(B) In a lattice $(L, \leq)$, if $a \leq b \leq c$, then verify that **6**

(i) $a \oplus b = b * c$

(ii) $(a * b) \oplus (b * c) = (a \oplus b) * (a \oplus c)$

OR

Define following :

(i) Complete Lattice

(ii) Complemented Lattice

(iii) Boolean algebra

(iv) Atoms in Boolean algebra

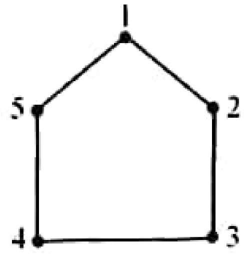
(v) Sub-Boolean algebra

(vi) Bounded Lattice

4\. (A) Attempt the following : **8**

(i) Define Complete graph. Draw a complete graph on seven vertices.

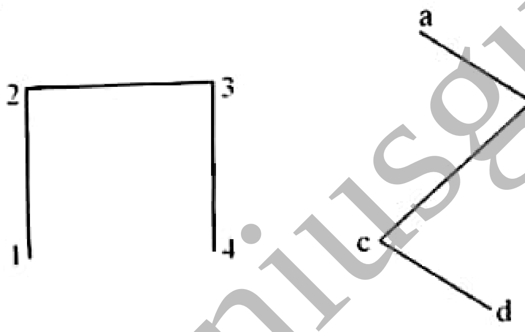
(ii) Define Complete of a graph. For a given graph below .obtain its complemented graph.



OR

Attempt the following : **8**

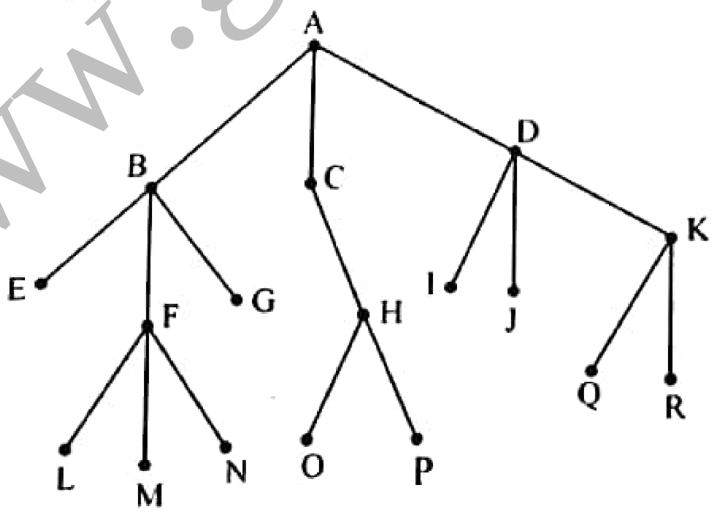
(i) Define the Isomorphic Graph. Check that the graphs given below are isomorphic or not ?



(ii) Define the following terminology :

Forest, Degree of a vertex, Node base, Multi Graph

(B) Convert the following tree into binary tree : **6**



OR

(B) Attempt the following :

(i) Draw the diagraph G corresponding to the following matrix :

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

Find total degree of all vertices. State which are even and which are odd vertices.

(ii) Give other three types of representation of the tree given below :

(A(B(E)(F))(C(G)(H)(I))(D(J(L)(M)(N))(K)))

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5. Do as directed :

(1) True/False : "Every binary operation defined on any set A having only one element is both commutative and associative."

(2) A group G in which $(ab)^2 = a^2b^2$ for all a, b in G is necessarily ________ .

(a) cyclic (b) abelian (c) finite (d) none of these

(3) ________ element has no multiplicative inverse in group <Z, ∗>.

(a) 0 (b) 1 (c) –1 (d) none

(4) If '∗' is defined on the set of real numbers R by $a * b = \sqrt{a^2 + b^2}$, the identify element of R with respect to '∗' is ________ .

(a) 0 (b) 1 (c) –1 (d) none

(5) A tree is graph with no ________ .

(a) loop (b) vertex (c) cycle (d) edge

(6) Is there a simple graph with degree sequence (0, 2, 2, 3, 4) ? [Yes/No]

(7) A vertex is said to be ________ vertex if its total degree is 7.

(a) even (b) odd (c) a & b both (d) none of these

(8) If one row of the incidence matrix of a graph has all entries 0, what information can you deduce about the graph ?

(9) Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by R = {(a – 1, a + 1)/a ∈ A}. Find the range of R.

(10) True/False : "{{x/x > 5}, {x/x < 5}} is a partition of the set of real numbers."

(11) Find the maximal and minimal elements of set P = {2, 3, 5, 7, 11, 13}, ordered by divisibility.

(12) True/False : "There is no horizontal line in a Hasse diagram of a POSET."

(13) In a Boolean algebra (S_{30}, ∗, ⊕, ', 0, 1), 5 ∗ 15 = ________ .

(14) Anti-atoms are immediate predecessor of greatest element ________ .

(a) 0 (b) 1