Seat No. : _______________

NB-104

November-2022

B.Sc., Sem.-V

301 : Mathematics
(Linear Algebra – II)

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Right side figures indicates marks of that question.

1. (A) Define an annihilator. Prove that if A is a subset of real vector space V then A° (annihilator of A) is a subspace of V*. 7

(B) Find the Dual basis of the basis $B_2 = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for the vector space R^3. 7

OR

(A) Prove that L(U,V) is a vector space. 7

(B) If a linear map $T : R^3 \to R^2$ is definded as $T(x, y, z) = (x + y, 2y + z)$; $(x, y, z) \in R^3$ then Solve the operator equation $T(x, y, z) = (2, 4)$. 7

2. (A) State and prove the Cauchy-Schwarz inequality. 7

(B) Prove that the following defines as inner product on R^2 :
$\langle u, v\rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$; where $u = (x_1, x_2)$, $v = (y_1, y_2) \in R^2$. 7

OR

(A) Prove that every orthogonal set in an inner product space is Linearly independent. 7

(B) Apply the Gram-Schmidt orthogonalization process to the basis
$B = \{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$ in order to get orthogonal basis for R^3. 7

3. (A) For matrix $A = (a_{ij}) \in M_n$, in usual notation prove that

$$\det A = \sum_{f \in S_n} (\operatorname{sgn} f) a_{f(1)1} a_{f(2)2}, \ldots a_{f(n)n}.$$ 7

(B) State (only) the Cramer's rule and using it solve $2x + y = 2$, $3y + z = 1$ and $4z + x = 5$. 7

OR

(A) In usual notation prove that det (AB) = det A . det B. 7

(B) Using the properties of determinants, prove that det A = 4abc.

$$\text{where } A = \begin{bmatrix} b+c & b & c \\ a & a+c & c \\ a & b & a+b \end{bmatrix}.$$ 7

4. (A) Prove that distinct Eigen vectors of $T \in L(U,V)$ co-responding to distinct Eigen values of T are Linearly independent. 7

(B) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}$ and hence find A^{-1}. 7

OR

(A) Prove that the eigen values of symmetric linear transformation are real. 7

(B) Let $A = \begin{bmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{bmatrix}$ be a real symmetric matrix. Find an orthogonal matrix P such that $D = P^{-1} AP$ is a diagonal matrix. 7

5. Answer in Short : (Attempt any **SEVEN**) **14**

(1) Define the Dual basis and Dual space.

(2) If linear transformation $T : R^3 \to R^2$ and $S : R^3 \to R^2$ defended as $T(x, y, z) = (x+ y, 2y+ z)$; and $S(x, y, z) = (x - 3y, 2y - z)$; $\forall(x, y, z) \in R^3$, then find T + S and 5S.

(3) Define Homogeneous and non-homogeneous Linear Operator.

(4) Define inner product on a vector space V and a norm of a vector in an inner product space.

(5) State (only) triangular inequality and parallelogram law.

(6) If $(R^2, <u, v>)$ is an inner product space, where $< u, v > = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$

$u = (x_1, x_2), v = (y_1, y_2) \in R^2$ then find the norm of vector $x = (5, 2)$.

(7) State (only) the Laplace's expansion for det A.

(8) If $f = \begin{pmatrix} 1234567 \\ 3451726 \end{pmatrix}; g = \begin{pmatrix} 1234567 \\ 2546173 \end{pmatrix} \in S_7$ then find fog, $(gof)^{-1}$.

(9) If $A = \begin{bmatrix} 1 & -5 & 3 \\ 2 & 8 & -4 \\ 3 & 4 & -2 \end{bmatrix}$ then find trace of matrix A and A^T.

(10) Define eigen values and eigen vectors of a linear operator.

(11) Define symmetric linear transformation.

(12) Define non-singular matrix.
