Seat No. : _______________

MB-110

March-2018

B.Sc., Sem.-V

CC-301 : Mathematics
(Linear Algebra-II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (i) Notations are usual everywhere.
(ii) All the questions are compulsory and carry of **14** marks.
(iii) The right hand side figures indicate marks of the question/sub-question.

1. (A) If $T : U \to V$ is linear map, $v_0 \in R(T)$ and if $T(u) = \bar{0}_V$ has a non-trivial solution $u_0 \neq \bar{0}_U$ then prove that $T(u) = v_0$ has an infinite solution set namely $u_0 + N(T)$. **7**

OR

Define a linear functional and show that the trace function $tra : \mu_{n,n} \to R$ defined as $traA = \sum_{i=1}^{n} a_{i,\,i,}$ for each matrix $A = (a_{ij}) \in \mu_{n,\,n}$ is linear function.

(B) If a linear map $T : V_3 \to V_3$ is defined as $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 + e_3$, $T(e_3) = e_3 + e_1$ then solve the operator equation $T(x_1, x_2, x_3) = (4, 6, 8)$. **7**

OR

Find the dual basis of the basis $B = \{(1, 2, 0), (0, 1, 2), (2, 0, 1)\}$ for the vector space V_3.

2. (A) Prove that a finite dimensional inner product space has an orthogonal basis. **7**

OR

State and derive the Cauchy-Schwarz inequality.

(B) If for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ the map $<,>$ is defined as

$<x, y> = y_1\,[x_1 - x_2] + y_2\,[2\,x_2 - x_1]$ then show that $<,>$ is an inner product on R^2. **7**

OR

Apply the Gram-Schmidt orthogonalization process on the set
$A = \{(0, 1, 1, 1), (1, 1, 0, 1), (1, 1, 1, 0)\}$ in order to get orthogonal set in V_4.

3. (A) If $i \neq j$, $\alpha \in R$ and if det : $V^n \rightarrow R$ is a function satisfy the expected properties of the determinant then prove the followings : **7**

(i) $\det(v_1, v_2, \ldots, v_i, \ldots, v_j, \ldots, v_n) = \det(v_1, v_2, \ldots, v_i + \alpha v_j, \ldots, v_j, \ldots, v_n)$

(ii) $\det(v_1, v_2, \ldots, x_j + y_j, \ldots, v_n) = \det(v_1, v_2, \ldots, x_j, \ldots, v_n) + \det(v_1, v_2, \ldots, y_j, \ldots, v_n)$

OR

State the prove the Cramer's rule for solving a system of linear equations.

(B) If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 4 & 5 \\ 2 & 3 & 1 & 4 \\ 1 & 0 & -1 & 3 \end{pmatrix}$ then find detA by applying the Laplace Expansion about the second row of the matrix A. **7**

OR

If $A = \begin{pmatrix} x+a & b & c & d \\ a & x+b & c & d \\ a & b & x+c & d \\ a & b & c & x+d \end{pmatrix}$ then compute detA without expansion.

4. (A) Express the characteristic equation of 2×2 matrix A in terms of Trace of A and det A. Also prove that a 2 x 2 real and symmetric matrix has only real eigen values. **7**

OR

State and prove the Cayley-Hamilton's Theorem.

(B) Diagonalize the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. **7**

Also find the modal matrix which diagonalizes A.

OR

Identify the quadric in R^3 given by $f(x, y, z) = 4xz + 4y^2 + 8y + 8 = 0$.

5. Answer any **seven** of the following questions in short : **14**

(1) Define a scalar multiple of a linear map and composition of linear maps.
(2) Define homogeneous and nonhomogeneous operator equations.
(3) State the dual basis existence theorem.
(4) State the triangle inequality.
(5) Define an orthogonal linear map.
(6) State any **two** of the three expected properties of the determinant mapping.
(7) Without expansion find detA if $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 5 & 4 & 3 \end{bmatrix}$.
(8) Define eigen value and eigen vector of an endomorphism.
(9) Define : (i) eigen basis (ii) Diagonalizable matrix.