Seat No. : _______________

NC-108

November-2022

B.Sc., Sem.-V

302 : Mathematics
(Analysis – I)

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Figures to the right indicate full marks of the questions.

1. (a) Let S be any non-empty, bounded subset of R and a be any real number then show that Sup(a + S) = a + SupS 7

(b) Prove that $\sqrt{11}$ is not a rational number. 7

OR

(a) State and prove Archimedean Property. Using this prove that if $S = \{1/n : n \in N\}$ then inf S = 0. 7

(b) Find all $x \in R$ that satisfies the inequality $4 < |x+2| + |x-1| < 5$. 7

2. (a) State and prove Sandwich Theorem. 7

(b) If $s_1 = \sqrt{5}$ and $s_{n+1} = \sqrt{5s_n}$ for $n \geq 1$, prove that (s_n) is a monotonic increasing sequence bounded above and $\lim_{n\to\infty} s_n = 5$. 7

OR

(a) State and prove Cantor's nested interval theorem. 7

(b) Let $x_n = \sum_{k=1}^{n} \frac{1}{k^3} = 1 + \frac{1}{2^3} + \frac{1}{3^3} + ... + \frac{1}{n^3}$. Show that sequence $\{x_n\}$ is a Cauchy sequence. 7

3. (a) Suppose that the function f is continuous on the interval $[a, b]$. $f(a) \neq f(b)$, and k is any number between $f(a)$ and $f(b)$. Then prove that there exists at least one point $c \in (a, b)$ such that $f(c)=k$. 7

(b) Show that any polynomial of odd degree must have at least one real root. 7

OR

P.T.O.

(a) Suppose that the function f is continuous on the interval $[a, b]$ then prove that f is uniformly continuous on [a, b]. 7

(b) Discuss the uniform continuity of the following functions : 7

(i) $f(x) = x^2$ on $[0, \infty)$

(ii) $g(x) = (x-1) / (x + 2)$ on $[0,\infty)$

4. (a) State and prove Darboux's Theorem. 7

(b) Verify Mean Value Theorem(MVT) for $f(x) = x(x-1)(x-2)$ on [0, 3]. 7

OR

(a) Suppose that the composition $f_0 g$ is defined in a neighbourhood of x_0 and that g is differentiable at x_0 and f is differentiable at $y_0 = g(x_0)$. Then prove that $f_0 g$ is differentiable at x_0, and $(f_0 g)'(x_0) = f'(g(x_0))g'(x_0)$. 7

(b) Evaluate : (1) $\lim\limits_{x\to\infty} \frac{(\ln x)^2}{x}$ (2) $\lim\limits_{x\to 0}\left(\frac{2^x+3^x+5^x}{3}\right)^x$ 7

5. Attempt any **Seven** short questions : 14

(i) If $A = \left\{\frac{n}{n+1} / n \in N\right\}$, then inf A=____.

(ii) If $B = \{x \in R / x^2 - 2x + 2 = 0\}$. then sup B =____.

(iii) Give example of countable proper subset of N.

(iv) Find the cluster points of the sequence $\{x_n\} = \{cos(n\pi/4)\}$

(v) Give an example of a sequence which is unbounded and oscillatory.

(vi) Obtain the limit if exists : $\lim\limits_{n\to\infty} \frac{2n+1}{5n-1}$.

(vii) Evaluate : $\lim\limits_{x\to 1^-} [x-3] + [x+5]$ if exist.

(viii) $\lim\limits_{x\to\infty} \frac{x-3}{x} = ?$

(ix) Give example of a function with removable discontinuity.

(x) State Rolle's Mean Value theorem.

(xi) Determine where the function $f(x) = |x^2 - 4|$ is not differentiable.

(xii) State L'Hospital's First Rule.
