Seat No. : _______________

ND-119

November-2022

B.Sc., Sem.-V

303 : Mathematics
(Complex Variables and Fourier Series)

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Write the question number in your answer sheet as shown in the question paper.
(3) Figures to the right indicate marks of the question / sub-question.

1. (i) Prove that (i) $\sin ix = i \sinh x$ (ii) $\cos ix = \cosh x$ and hence separate $\tanh (x + iy)$ into its real and imaginary parts. 7

(ii) Prove that for any rational number n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + 1\, i\sin n\theta$. 7

OR

(i) If a, a^2, a^3, a^4 are the roots of $x^5 = 1$, then find them and show that $(1 - a)(1 - a^2)(1 - a^3)(1 - a^4) = 5$. 7

(ii) Define convergence of the series of complex numbers. Suppose that $z_n = x_n + iy_n$, $n \in \mathbb{N}$ and $S = X + iY$, then prove that $\sum_{n=1}^{\infty} z_n = S$ if and only if $\sum_{n=1}^{\infty} x_n = X$ and $\sum_{n=1}^{\infty} y_n = Y$. 7

2. (i) State and prove necessary conditions for a function $f(x, y) = u(x, y) + iv(x, y)$ to be analytic at a point in a region R. 7

(ii) Show that $f(z) = \cos z$ is an entire function and also find its derivative. 7

OR

(i) If $f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z} ; & z \neq 0 \\ 0 ; & z = 0 \end{cases}$ then, check whether component functions satisfy C-R equations at origin. Also check if f(z) is analytic at origin. 7

(ii) Derive Cauchy - Riemann equations in polar form. Hence prove that $r^2 u_{rr}(r, \theta) + r u_r(r, \theta) + u_{\theta\theta}(r, \theta) = 0$. 7

3. (i) If f(z) is analytic at a point z_0 of the domain then prove that mapping w = f(z) is conformal at z_0 iff $f'(z_0) \neq 0$. 7

(ii) Show that the transformation $w = \frac{1}{z}$, $z \neq 0$ transforms lines and circles to lines and circles. 7

OR

(i) Show that the transformation $w = \frac{2z+3}{z-4}, z \neq 4$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line 4u + 3 = 0 and explain why the curve obtained is not a circle. 7

(ii) Consider the map w = 2(1 + i)z. Find image R' in w plane of the rectangular region R bounded by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 2$ in z plane. Sketch both R and R'. 7

4. (i) State and prove Bessel's inequality. 7

(ii) Find Fourier series expansion of $f(x) = x^3, -\pi < x < \pi$. 7

OR

(i) State and prove Riemann-Lebesgue theorem. 7

(ii) Find Fourier series expansion of $f(x) = \begin{cases} 1+x, & 0<x<\pi \\ 1-x, & \pi<x<2\pi \end{cases}$ 7

5. Give the answer in brief : (Any **Seven**) 14

(1) Plot all the roots of $z^3 = 1$.

(2) State triangle inequality.

(3) Evaluate $ln\,(2i)$.

(4) Define an Analytic function.

(5) Define harmonic function.

(6) Can there be a function which is differentiable but not analytic ? If so give one example (without justification).

(7) Define Conformal transformation.

(8) Define Bilinear transformation.

(9) Find fixed points of $w = \frac{z+1}{3z-4}, z \neq \frac{4}{3}$.

(10) Define Periodic function.

(11) Find Fourier coefficient a_0 for the function $f(x) = |x|$ in (–2, 2).

(12) Find Fourier series expansion for the function $f(x) = \cos x$ in $(0, 2\pi)$.