Seat No. : _____

NE-107

November-2022

B.Sc., Sem.-V

304 : Mathematics
(Mathematical Programming)

Time : 2½ Hours] [Max. Marks : 70

Instructions : (1) **All** questions are compulsory.
(2) Right side indicates marks of that question.

1. (A) Let $K \subset R^n$, prove that K is convex set if and only if every finite convex linear combination of elements in K is also belongs to K. 7

(B) A company has two grades of inspectors 1 and 2, who are to be assigned for a quality control inspection ? It is required at least 2000 pieces be inspected per 8-hour day. A Grade 1 inspector can check pieces at the rate of 40 per hour, with an accuracy of 97%. A Grade 2 inspector checks at the rate of 30 pieces per hour with an accuracy of 95%. The wage rate of a Grade 1 inspector is ₹ 5 per hour while that of a Grade 2 inspector is ₹ 4 per hour. An error made by an inspector costs ₹ 3 to the company. There are only nine Grade 1 inspectors and eleven Grade 2 inspectors so as to minimize the total cost of the inspection. Forumulate this problem as a linear programming model. 7

OR

(A) Define convex set. Prove that $X = \{(x_1, x_2) / 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. 7

(B) A company produces three products A, B and C. These products require three ores O_1, O_2 and O_3. The maximum quantities of the ores O_1, O_2 and O_3 (available are 22 tonnes, 14 tonnes and 14 tonnes respectively. For one tonne of each of these products, the ore requirements are : 7

	A	B	C
O_1	3	–	3
O_2	1	2	3
O_3	3	2	3
Profit per tonne (₹ in thousand)	1	4	5

The company makes a profit of ₹ 1,000, 4,000 and 5,000 on each tonne of the products A, B and C respectively. Forumulate this problem as a linear programming model.

2. (A) Explain Simplex algorithm for solving linear programming problem. 7

(B) Solve the following LPP by simplex method : 7

Use the two-phase method to

Min. $Z = x_1 + x_2 + x_3$

Subject to $x_1 - 3x_2 + 4x_3 = 5; x_1 - 2x_2 \leq 3; 2x_2 - x_3 \geq 4$

$x_1, x_2 \geq 0, x_3$ is unrestricted.

OR

(A) Explain Gomory's cutting plane method for solving Integer programming. 7

(B) Using appropriate simplex method to solve : 7

Max. $Z = 2x_1 + 3x_2 + 5x_3$

Subject to $3x_1 + 10x_2 + 5x_3 \leq 15; 33x_1 - 10x_2 + 9x_3 \leq 33; x_1 + 2x_2 + x_3 \geq 4;$

$x_1, x_2, x_3 \geq 0$

3. (A) Prove that the value of the objective function f(x) for any feasible solution of the primal is not less than the value of the objective function g(y) for any feasible solution of the dual. 7

(B) Apply the principle of duality to solve the following LPP : 7

Min. $Z = 2x_1 + 2x_2$

Subject to the constraint

$2x_1 + 4x_2 \geq 1; x_1 + 2x_2 \geq 1; 2x_1 + x_2 \geq 1; \quad x_1, x_2 \geq 0$

OR

(A) Explain Primal-Dual Relationship. Prove that Dual of the Dual is Primal. 7

(B) Using dual simplex method to solve the following Linear Programming Problem : 7

Max. $Z = -3x_1 - 2x_2$

$x_1 + x_2 \geq 1; x_1 + x_2 \leq 7; x_1 + 2x_2 \geq 10; x_2 \leq 3; \quad x_1, x_2 \geq 0$

4. (A) Explain : 7

(1) Loop in transportation problem.

(2) Degeneracy in transportation problem.

(B) Suppose that there are six people applying for five jobs; and it is desired to fill each job with exactly one person. The costs for filling the jobs with six people are given in the following table : 7

	J1	J2	J3	J4	J5
P1	27	23	22	24	27
P2	28	27	21	26	24
P3	28	26	24	25	28
P4	27	25	21	24	24
P5	25	20	23	26	26
P6	26	21	21	24	27

Determine the optimal assignment plan. i.e. the plan whereby the cost of assigning the people is minimized.

OR

(A) Explain Hungarian method to solve Assignment Problem. 7

(B) A product is manufactured by four factories A, B, C and D. The unit production costs in them are ₹ 2, ₹ 3, ₹ 1 and ₹ 5 respectively. Their production capacities are 50,70,30 and 50 units respectively. These factories supply the product to four stores, demands of which are 25,35,105 and 20 units respectively. Unit transportation cost in rupees from each factory to each store is given in the table below. 7

		STORES			
		1	2	3	4
FACTORIES	A	2	4	6	11
	B	10	8	7	5
	C	13	3	9	12
	D	4	6	8	3

Determine the extent of deliveries from each of the factories to each of the stores so that the total production and transportation cost is minimum.

5. Answer in short : (Attempt any **SEVEN**) **14**

(1) Define extreme point.

(2) Give two examples of non-convex set.

(3) Define convex linear combination.

(4) Define Artificial variable.

(5) How we solve any Linear Programming Problem using Two-Phase simplex method ?

(6) When we say any Linear Programming Problem has Unbounded solution ?

(7) Give one advantage of Duality.

(8) Explain in short dual simplex method.

(9) How we get Primal solution without solving it ?

(10) Explain North-West Corner Rule.

(11) How do we solve unbalanced transportation problem ?

(12) How do we solve unbalanced Assignment Problem ?
