Seat No. : _______________

ME-108

March-2018

B.Sc., Sem.-V

CC-304 : Mathematics

(Mathematical Programming)

Time : 3 Hours] **[Max. Marks : 70**

Note : Each questions carry equal marks

1. (a) Define convex hull. Prove that convex hull is a convex set. **7**

OR

Prove that intersection of two convex sets is a convex set. Is union convex set ? Justify your answer.

(b) Let $C_1 = \{ x \in R^n : ||x|| \leq 3\}$, $S_2 = \{ x \in R^n : ||x|| \geq 3\}$. Determine convexivity of S_1 and S_2. **7**

OR

A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a standard LPP.

2. (a) Prove that every basic feasible solution of an LPP is a vertex of the set of all feasible solution of an LPP. **7**

OR

Prove that the set of all feasible solutions of an LPP is a closed convex set which bounded below.

(b) Solve the following LPP by Simplex method. **7**

Max $Z = 2x_1 + 4x_2 + 3x_3 + x_4$

s.t. $x_1 + 3x_2 + x_4 \leq 4,$

$2x_1 + x_2 \leq 3,$

$x_1 + 4x_3 + x_4 \leq 3$

$x_j \geq 0, j = 1, 2, 3, 4.$

OR

Solve the following LPP by Big-M method.

Min $Z = 5x_1 + 2x_2 + 10x_3$

s.t. $x_1 - x_3 \leq 10,$

$x_2 + x_3 \geq 10,$

$x_j \geq 0, j = 1, 2, 3.$

3. (a) State and prove fundamental theorem of duality. **7**

OR

Define dual of an LPP. Prove that dual of dual is primal LPP.

(b) Solve the following LPP by dual Simplex method. **7**

Min $Z = 2x_1 + x_2 + x_3$

s.t. $4x_1 + 6x_2 + 3x_3 \leq 8,$

$-x_1 + 9x_2 - x_3 \geq 3,$

$2x_1 + 3x_2 - 5x_3 \geq 4$

$x_j \geq 0, j = 1, 2, 3.$

OR

By principle of duality solve the following LPP.

$$\text{Max } Z = 3x_1 + 2x_2$$

$$\text{s.t. } 2x_1 + x_2 \leq 5,$$

$$x_1 + x_2 \leq 3,$$

$$x_j \geq 0, \text{ j} = 1, 2.$$

4. (a) Explain the formulation of a transportation problem. **7**

OR

Prove that every balanced transportation problem has a triangular basis.

(b) Find the minimum transportation cost for the following transportation problem by MODI method. **7**

	D_1	D_2	D_3	D_4	a_i
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
b_j	7	5	3	2	17

OR

Solve the following assignment problem.

	A	B	C	D
I	40	35	38	41
II	42	35	34	40
III	38	34	34	37
IV	39	36	38	36

5. Attempt any **seven** in short : **14**

(1) Define surplus variable.

(2) Define solution of an LPP.

(3) Define infeasible solution in LPP.

(4) Every transportation problem is an LPP. (True/False)

(5) Define triangular basis in a transportation problem.

(6) Find the dual of the following LPP.

$$\text{Min } Z = x_1 + 2x_2$$

$$\text{s.t. } 2x_1 + 4x_2 \leq 16,$$

$$x_1 - x_2 = 3,$$

$$x_1 \geq 10$$

$$x_j \geq 0, \ j = 1, 2.$$

(7) Find the initial solution of the following transportation problem.

	D_1	D_2	D_3	D_4	a_i
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
b_j	5	8	7	14	

(8) How many numbers of basic variables in n × n type of assignment problem ?

(9) Every assignment problem is transportation problem. (True/False)