Seat No. : _____

NF-115

November-2022

B.Sc., Sem.-V

305 : Mathematics

(A : Discrete Mathematics)

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (1) **Q-4** is compulsory question of short questions.
(2) Notations are usual, everywhere.
(3) Figures to the right indicate marks of the question/sub-question.

1. (A) Let $(L, \leq)$ be Lattice, $a, b \in L$. Prove that
$a \leq b \Leftrightarrow a \vee b = b \Leftrightarrow a \wedge b = a$ **9**

(B) Answer the following questions : **9**

(i) Consider the relation $R = \{(i, j) : | i - j | = 2\}$ on $\{1, 2, 3, 4, 5, 6\}$ is R Equivalence relation.

(ii) Let $X = (2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $a \leq b$ if a Divides b. Draw the Hass diagram of $(X, \leq)$

OR

(A) Answer the following questions : **9**

(i) State and prove Modular Inequality in Lattice.

(ii) State and prove Isotonicity Property in Lattice.

(B) Answer the following questions : **9**

(i) Show that the similarity of Matrices of $n \times n$ matrices is an Equivalence relation.

(ii) Show that (S_6, D) is Lattice.

2. (A) Prove that Direct Product of Two Lattices is also Lattice. **9**

(B) Show that the Lattice (S_n, D) for $n = 36$ is isomorphic to the direct product of lattices for $n = 8$ and $n = 27$. **9**

OR

(A) In a complemented distributive lattice the following are equivalent : **9**

(1) $a \leq b$ (2) $a \wedge b' = 0$

(3) $a' \vee b = 1$ (4) $b' \leq a'$

(B) Answer the following questions : **9**

(i) Draw the Hass diagram of $(L_3, \leq_3,)$ where $L = \{0, 1\}$.

(ii) Find out complement of each element of the lattice $(S_{30}, *, \oplus)$.

3. (A) Let (B, *, $\oplus$, ', 0, 1) be Boolean Algebra then prove following : 9

(1) $x_1 \leq x_2 \Rightarrow A(x_1) \subset A(x_2)$

(2) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(3) (2)A $(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(4) $A(x') = A - A(x)$

(5) $A(x) = A(y) \Leftrightarrow x = y$

(B) Answer the following questions : 9

(i) If B = {0, 1} show that (B, *, $\oplus$, ', 0, 1) is a Boolean Algebra.

(ii) In any Boolean Algebra show that $a = 0 \Leftrightarrow ab' + a'b = b$.

OR

(A) State and prove Stone Representation Theorem. 9

(B) Answer the following questions : 9

(i) Obtain sum of product canonical form in three variables $x_1 * x_2$.

(ii) Obtain sum of product canonical form in three variables $x_1 \oplus x_2$.

4. Answer any **eight** in short : 16

(1) Define : Inverse Relation

(2) If R and S are equivalence relation on the set X. Then R $\cup$ S is always an equivalence relation ? Justify your answer.

(3) X = (2, 3, 6, 12, 24, 36} and the relation $\leq$. A = (2, 3, 6} then find upper bound and lower bound.

(4) State Absorption law in Lattice.

(5) Define : Lattice as an Algebraic System

(6) Define : Bounded Lattice

(7) Define : Sub Lattice

(8) Define : Chain

(9) Define : Minterm

(10) Define : Maxterm

(11) Define : Atom

(12) Define : Sub Boolean Algebra

Seat No. : _____

NF-115

November-2022

B.Sc., Sem.-V

305 : Mathematics

(B : Number Theory)

Time : 2½ Hours] **[Max. Marks : 70**

Instruction : All questions are compulsory.

1. (a) State and prove the necessary condition to solve linear Diophantine equation $ax + by = c$, also derive the general solution if it has solution. 9

   (b) (i) Prove : $21 \mid 4^{n+1} + 5^{2n-1}, \forall\, n \geq 1$. 9

   (ii) Prove that if a & b are both odd integers then $16 \mid a^4 + b^4 - 2$.

OR

   (a) State and prove the Division Algorithm theorem for integers. 9

   (b) Solve in N : 9

   (i) $172x + 20y = 1000$

   (ii) $56x + 72y = 40$

2. (a) Show that every positive integer $n > 1$ can be expressed as a product of primes in unique way. 9

   (b) Define the complete set of residues modulo n. Are the sets $0, 1, 2, 2^2, ..., 2^9$ and $0, 1^2, 2^2, 3^2, ..., 10^2$ form a complete set of residues modulo 11 ? Justify. 9

OR

   (a) Show that the quadratic congruence $x^2 + 1 \equiv 0 (\text{mod } p)$, p = prime > 2 has solution if and only if $p \equiv 1 (\text{mod } 4)$. 9

   (b) (i) Using Chinese Remainder Theorem, solve : $17x \equiv 9 \text{ (mod } 2\,76)$. 9

   (ii) If 495 divides the integer $273\ x\ 49y5$ then find the missing digits x & y.

3. (a) State and prove the Wilson's theorem. **9**

Also verify that $4(29!) + 5!$ is divisible by 31 or not.

(b) (i) State the Euler's Theorem and prove that $a^{1729} \equiv a \pmod{1729}$, $a \in Z$. **9**

(ii) Find the least pseudo prime and absolute pseudo prime numbers.

OR

(a) State & prove the Fermat's little theorem. **9**

Is converse of Fermat's little theorem is true ? Justify.

(b) (i) Prove: $p \mid a^p + a(p-1)!$ & $p \mid (p-1)!a^p + a$ " prime p & a Î N. **9**

(ii) What is the unit digit of 9^{9^9} ?

4. Attempt any **eight** in short : **16**

(1) Find a prime of the form $n^3 - 1$.

(2) Find a prime P such that the numbers $p^2 + 8$ and $p^3 + 4$ are also primes.

(3) Find the remainder when $\sum_{n=1}^{25}(n!)$ is divided by 24.

(4) Find the remainder when 2021^{2022} is divided by 2020.

(5) Find the missing digits X, Y, Z if 396 divides the number 3X6Y9Z.

(6) Find all prime numbers that divide 49 !.

(7) Find $\phi(5040)$ & $\phi(1001)$.

(8) 50! End with how many zeroes ?

(9) Find the unit digit of 5^{81}.

(10) Is 1111998899 divisible by 7, 11 and 13 ? Justify.

(11) Find possible n such that n! +1 is perfect square, where $0 \leq n \leq 10$.

Seat No. : _____

NF-115

November-2022

B.Sc., Sem.-V

305 : Mathematics

(Financial Mathematics)

Time : 2½ Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.

(2) Notations are usual, everywhere.

(3) Figures to the right indicate marks of the question/sub-question.

1. (a) Write a short note on Interest rates. (Discuss annual, semi-annual, quarterly, monthly, weekly, daily compounding etc.) 9

(b) What is the Present value of ₹ 25,00,000 received after ten years, for opportunity cost (interest rate) is 5.5% per year compounded annually, semi-annually, quarterly, monthly, weekly, daily and continuously. 9

OR

(a) Define bonds, shares & index, also define arbitrage with example. 9

(b) What is the Future value of ₹ 2,05,000 invested for 7 years, for opportunity cost (interest rate) is 7.5% per year compounded quarterly, weekly, daily, continuously, also find effective rate of interest in each case. 9

2. (a) Write a short note on Bond, also define bond redemption at Par, at premium and at discount. 9

(b) Consider a monthly coupon bond with annual coupon rate 13%, with the face value ₹ 1,000 for 3-year tenure consider the annual rate of interest in effect is 24%. Find the NPV for this bond. 9

OR

(a) Show that for a bond of n years with annual coupon payment C and face value F, if its yield (yield to maturity) is λ then its price is given by

$$P = \frac{1}{(1+\lambda)^n}\left[\frac{(1+\lambda)^n - 1}{\lambda}C + F\right].$$ 9

(b) Consider a portfolio with two bonds A and B, immunize this bonds portfolio using Macaulay duration, if the amount to be invested in portfolio P = 2,00,000 /- and for the duration of 3 years. Here Bond A is zero coupon bond of 2 years and Bond B is of 5 years annual coupon bond with annual coupon payment 300 and the face value of 1000 with the desired yield to maturity 8% continuously compounded. Then determine the amount of money to be invested in each bond. 9

3. (a) Write the Markowitz portfolio optimization problem with short selling and derive constrained optimization method using Langrage's multiplier. 9

(b) Calculate the portfolios mean return and variance using the following details :

$R = (0.39, 1.16, -0.59)^T$, $W = (0.3, 0.2, 0.5)$ and $CV = \begin{bmatrix} 1.02 & 1.14 & 0.29 \\ 1.14 & 2.20 & 0.60 \\ 0.29 & 0.60 & 1.32 \end{bmatrix}$ find

$\bar{r}$ & σ^2 for portfolio. 9

OR

(a) Write a short note on portfolio diagram and choice of asset. 9

(b) Consider a portfolio of three assets, A, B & C with the following properties : 9

$\bar{r}_A = 0.4$, $\bar{r}_B = 0.3$, $\bar{r}_C = 0.7$,

$\sigma_A = \sigma_B = \sigma_C = 1$ & $\sigma_{AB} = \sigma_{BC} = \sigma_{AC} = 0$

For fixed $\bar{r} = 0.7$ find the minimum variance portfolio.

4. Attempt any **eight** of the followings in short : **16**

(1) Define an Asset.

(2) Define return and rate of return.

(3) Write types of financial instrument.

(4) Define an Arbitrage.

(5) Define Net Present Value of given cash flow.

(6) Define perpetuity.

(7) Define Internal Rate of Return.

(8) Define Quasi-Modified Duration for annual discrete compounding.

(9) Define efficient frontier.

(10) Write the statement of one fund theorem.

(11) Write the statement of two fund theorem.

(12) Define short selling.