Seat No. : _______________

MF-107

March-2018

B.Sc., Sem.-V

SE-305 : Mathematics
(Number Theory)

Time : 3 Hours] **[Max. Marks : 70**

1. (a) State and prove the Division Algorithm theorem for integers. **9**

OR

Prove that for given integers a and b, not both of which are zero, there exist integers x and y such that gcd (a, b) = ax + by.

(b) (i) Prove that for positive integers a and b, gcd (a, b) lcm(a, b) = ab. **9**

(ii) Use the Euclidean Algorithm to obtain integers x and y satisfying gcd(1769, 2378) = 1769x + 2378y.

OR

Prove that the linear Diophantine equation ax + by = c has a solution if and only if d/c, where d = gcd (a, b). Further prove that if x_0, y_0 is any particular solution of this equation, then all other solutions are given by

$x = x_0 + \left(\frac{b}{d}\right)t$, $y = y_0 - \left(\frac{a}{d}\right)t$, where t is an arbitrary integer.

2. (a) Prove that there is an infinite number of primes. **9**

OR

(i) Define prime numbers. For any integers n, a and b, does n/ab implies n/a or n/b ? If not when this will hold true ?

(ii) Employing the Sieve of Eratosthenes, obtain all the primes between 100 and 200.

(b) Let n > 1 be fixed and a, b, c, d be arbitrary integers. Then prove that the following properties hold : **9**

(i) If a ≡ b (mod n) and c ≡ d (mod n), then a + c = b + d (mod n) and ac = bd (mod n).

(ii) If a ≡ b (mod n), then a + c ≡ b + c (mod n) and ac ≡ bc (mod n).

(iii) If a ≡ b (mod n), then ak ≡ bk (mod n) for any positive integer k.

OR

(i) Solve : 25x ≡ 15 (mod 29).

(ii) Solve following set of simultaneous congruences :
x ≡ 2 (mod 3), x ≡ 3 (mod 5), x ≡ 2 (mod 7)

3. (a) State and prove Fermat's Theorem. Is the converse of Fermat's theorem true ? Justify. **9**

OR

Show that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.

(b) State and prove Wilson's theorem. Is the converse of Wilson's theorem true ? Justify. **9**

OR

Show that Euler's Phi-function is a multiplicative function.

4. Answer the following in short : (Attempt any **eight**) **16**

(i) Explain the Euclidean algorithm.

(ii) State the Fundamental Theorem of Arithmetic.

(iii) State Euler's theorem.

(iv) State the Binomial Theorem.

(v) Explain canonical form of a number giving suitable example.

(vi) Find $\phi(450)$.

(vii) What is the remainder when 24^3 is divided by 13 ?

(viii) Find all prime numbers that divide 50!.

(ix) Write any two properties of Euler phi function.

(x) Prove or Disprove: If a/b and c/d, then ac/bd.