NC-108

November-2021

B.Sc., Sem.-V

CC-302 : Mathematics
(Analysis – I)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) There are total **9** questions.

(2) Attempt any **3** questions from first **8** question.

(3) Questions number **9** is compulsory.

(4) Notations and terminologies are standard.

1. (a) Let A be any set. Prove that there is no surjection of A onto the set P(A) of all subsets of A. **7**

   (b) Prove that there exists $x \in R$ such that $x^2 = 2$. **7**

2. (a) State and prove rational density theorem. **7**

   (b) Define lub of a set. Let A be a non-empty bounded subset of R. Define $\alpha A = \{\alpha a : a \in A\}$, where $\alpha > 0$. Prove that lub $(\alpha A) = \alpha \cdot$ lub A. **7**

3. (a) Let $\{x_n\}$ and $\{y_n\}$ be two convergent sequences such that

   $\lim_{n\to\infty} x_n = l$ and $\lim_{n\to\infty} y_n = m$. Prove that $\lim_{n\to\infty} x_n \cdot y_n = l \cdot m$. **7**

   (b) Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is convergent. **7**

4. (a) Prove that every Cauchy sequence of real numbers is convergent. **7**

   (b) Define Cauchy sequence. Prove that the sequence $\left\{\sum_{k=1}^{n} \frac{1}{k^3}\right\}$ is a Cauchy sequence. **7**

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5. (a) Let $f : E \subset \mathbb{R} \rightarrow \mathbb{R}$ be a function and $c \in \mathbb{R}$. Prove that f is continuous at c iff for every sequence $\{x_n\}$ in E with $x_n \neq c$, $\forall n \in \mathbb{N}$, $x_n \rightarrow c$, as $n \rightarrow \infty$, then $f(x_n) \rightarrow f(c)$, as $n \rightarrow \infty$. 7

(b) Prove that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, \infty)$ and uniformly continuous on $[c, \infty)$, $c > 0$. 7

6. (a) Let $f : [a, b] \rightarrow \mathbb{R}$ be a continuous function. Prove that f is uniformly continuous on $[a, b]$. 7

(b) Define $f : (0, 1) \rightarrow \mathbb{R}$ by $f(x) = \sin\left(\frac{2\pi}{x}\right)$, $x \in (0, 1)$. Discuss the uniform continuity of f on $(0,1)$. 7

7. (a) State and prove Inverse function theorem for derivative. 7

(b) Prove that the equation $x^3 - 3x^2 + b = 0$ has at most one root in the interval $[0,1]$. 7

8. (a) State and prove Darboux's Mean Value Theorem. 7

(b) Show that $\cos x = x^3 + x^2 + 4x$ has exactly one root in $\left[0, \frac{\pi}{2}\right]$. 7

9. Attempt any **four** : (in short) 8

(1) By definition prove that $lub\left\{1 - \frac{1}{n+3} : n \in \mathbb{N}\right\} = 1$.

(2) Define : Ordered field.

(3) Give an example of a sequence which is bounded but not convergent.

(4) State Extreme Value Theorem.

(5) By definition prove that $\lim_{n \to \infty} \frac{1}{n+1} = 0$.

(6) Define : Removable discontinuity.

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