

ND-139

November-2021

B.Sc., Sem.-V

303 : Mathematics
(Complex Variables & Fourier Series)

Time : 2 Hours]

[Max. Marks : 50

- Instructions : (1) All Questions in SECTION – I carry equal marks.
(2) Attempt any **THREE** questions in SECTION – I.
(3) Question – 9 in SECTION – II is **COMPULSORY**

SECTION – I

1. (A) State and prove De-Moivre's theorem. 7
(B) Find the cube root of $-27i$ 7
2. (A) Find all values of $(-1 + i)^{2/5}$. 7
(B) Find the radius of convergence of the series
(i) $\sum \frac{z^n}{n^n}$ (ii) $\sum \frac{n!z^n}{n^n}$
3. (A) State and prove sufficient condition for Cauchy-Riemann condition in Cartesian coordinates. 7
(B) If $u = x^3 - 3x^2y$ then prove that u is a Harmonic function and find the harmonic conjugate of u , also find analytic function $f(z)$. 7
4. (A) Show that $u(x, y) = e^{-x} \cos y$ is harmonic function then find its harmonic conjugate and function $f(x, y) = u + iv$. 7
(B) Define limit of a complex function $f(z)$. Prove that if the limit exists then it is unique. 7
5. (A) For $w = Ze^{i\frac{\pi}{4}}\sqrt{2}$, determine the region R of w – plane corresponding to the Rectangular region bounded by the lines $x = 0, y = 0, x = 2, y = 3$ in Z plane. 7
(B) State and Prove the necessary and the sufficient condition for the transformation to be Conformal. 7

- 6 (A) Under the transformation $w = 1/z$ find the image of $|z-2i| = 2$. Sketch the region in both the plane. 7
- (B) Find the image of the curve $|Z + i| = 2$ under the mapping $W = \frac{Z+i}{Z-i}$. 7
- 7 (A) If the series $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ converges uniformly to f on $[-\pi, \pi]$. Then prove that it is a Fourier Series for f on $[-\pi, \pi]$. 7
- (B) Find the Fourier Series to represent e^{ax} in the interval $[-\pi, \pi]$. 7
- 8 (A) If $f(x)$ is Riemann Integrable in $(-\pi, \pi)$ then the series $\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$ converges Where a_n, b_n are the Fourier Coefficients of $f(x)$. 7
- (B) Find the Fourier Series to represent $f(x) = x + x^2$ in the interval $[-\pi, \pi]$. 7

SECTION - II

9. Attempt any **FOUR** : 8
- (i) Express $-\sqrt{3} + i$ in the polar form.
- (ii) Find the value of $(1-i)^{14}$.
- (iii) Is $w = e^z$ entire? Justify.
- (iv) Examine the continuity of the function $\frac{Z}{Z-1}$
- (v) Find the point at which the mapping $W = Z^2 + \frac{1}{Z^2}$ is not conformal.
- (vi) Find $\int_{-\pi}^{\pi} \sin mx \cos nx dx$.