Seat No. : _______________

DD-128

December-2018

M.Sc., Sem.-I

404 : Mathematics
(Ordinary Differential Equations)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Answer the following questions : **14**

(1) Find the general solution of the equation $(1 + x^2)y'' + 2xy' - 2y = 0$ near $x = 0$.

(2) Define the radius of convergence of the power series $\sum_0^\infty a_n x^n$. Give three power series whose radius of convergence are $\frac{1}{5}$, 0 and ∞ respectively.

OR

(1) If f is analytic at x_0, prove that $f^{(n)}(x_0)$ exists for all n. Is the converse true? Justify.

(2) Find the general solution of the equation $4y'' + 4xy' + 4y = 0$ near $x = 0$.

(B) Attempt any **Four** : **4**

(1) Find the general solution of $y'' - 5y' + 6y = 0$.

(2) Solve the equation $y'' + 4y = 3 \sin x$.

(3) If f and g are analytic at x_0, prove that f + g is analytic at x_0.

(4) Find the differential equation satisfied by the family of circles with centres at (0, 0).

(5) Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ such that f' exist but f' is not continuous.

(6) Define an ordinary point of the equation $y'' + Py' + Qy = 0$.

2. (A) Answer the following questions : **14**

(1) Find two independent Frobenius solutions of the equation $xy'' + 2y' + xy = 0$.

(2) Show that $\tan^{-1} x = xF(\frac{1}{2}, 1, \frac{3}{2}, -x^2)$.

OR

(1) Solve the Euler equation $y'' + \frac{4}{x}y' + \frac{2}{x^2}y = 0$ near $x = \infty$.

(2) Solve the equation $(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$ near $x = 3$.

(B) Attempt any **Four** : **4**

(1) Define singular point and regular singular point by illustrations.

(2) Express the function $f(x) = \log(x + 1)$ in terms of the Legendre function $F(a, b, c, x)$.

(3) When we say that three functions f, g and h are linearly independent ? Give three such functions on the closed interval [0, 1].

(4) When we say that $x = \infty$ is an ordinary point of the equation $y'' + Py' + Qy = 0$?

(5) Give an equation which has a regular singular point at $x = \infty$.

(6) Give hypergeometric series. Why the series is called 'hyper' ?

3. (A) Answer the following questions : **14**

(1) State and prove the Rodrigues' formula.

(2) State and prove the Minimax property of Chebyshev polynomials.

OR

(1) State and prove the Least squares approximation method.

(2) State and prove the orthogonality of the Legendre polynomials $P_n(x)$.

(B) Attempt any **Three**. **3**

(1) Define Hermite polynomials.

(2) Find the first three terms of the Legendre series of the function $f(x) = e^x$.

(3) Show that $P_n(x)$ is an even function if n is even.

(4) Can we have a Legendre polynomial $P_n(x)$ such that $P_n(k) = 0$ for each $k \in N$? Justify.

(5) Prove that $P_n(-1) = (-1)^n$.

4. (A) Answer the following questions. **14**

(1) Define the Bessel function $J_p(x)$. Show that for any integer m, $J_{m+\frac{1}{2}}(x)$ is an elementary function.

(2) Explain in detail (with a simple illustration) the method of successive approximations.

OR

(1) Show that the functions $J_p(\lambda_n x)$ are orthogonal with respect to the weight function x on the closed interval [0, 1].

(2) State (without proof) Picard's theorem. Can we omit the continuity of $\frac{\partial f}{\partial y}$? Justify.

(B) Attempt any **Three**. **3**

(1) If $f : \mathbb{R} \to \mathbb{R}$ satisfies the inequality $|f(x) - f(y)| \leq M|x - y|$ for all $x, y \in \mathbb{R}$, what can be said about the continuity and differentiability of f ?

(2) Show that $|\sin x - \sin y| \leq x - y$ for all $x, y \in \mathbb{R}$.

(3) Show that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$.

(4) Find the value of $\left(\frac{7}{2}\right)!$

(5) Define elementary function and special function. Give two examples of each.