Seat No. : ______________

AB-110

April -2018

B.Sc., Sem.-VI

CC-307 : Mathematics

(Abstract Algebra – II)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry **14** marks.

(2) Figures to the right indicate marks of the question/sub-question.

(3) Notations are as usual.

1. (a) Define a ring. If R is a ring with unity and $a \in R$ then prove that **7**

(i) $(-1)\,a = -a$

(ii) $(-1)\,(-1) = 1$

OR

Prove that a field is an integral domain. Is converse true ? Justify your answer.

(b) Define characteristic of a ring. **7**

Prove that the characteristic of an integral domain is either prime number or zero.

OR

Prove that the set $F = \{a + b\sqrt{2} \ / \ a, b \in \mathbb{Q}\}$ is a Field.

2. (a) State and prove the fundamental theorem of homomorphism on rings. **7**

OR

Define 'Principal ideal'.

Prove that the ring $(\mathbb{Z}, +, \cdot)$ of all integers is the principal ideal ring.

(b) Check whether the set $A = \left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} / a, b \in \mathbb{Z}\right\}$ is a Subring of ring $R = (M_2(\mathbb{Z}); +; \cdot)$ or not ? **7**

OR

Prove that a field has no proper ideal.

3. (a) Find the G.C.D. of polynomials. **7**

$f(x) = x^3 - 2x^2 + 3x - 7$ and $g(x) = x^2 + 2$ over the field R. Express the G.C.D. as a linear combination of two polynomials.

OR

State Eisenstein criterion for irreducibility of polynomials.

Discuss the irreducibility of the polynomial $8x^3 + 6x^2 - 9x + 24$ over Q.

(b) Obtain all rational roots of the equation $2x^4 + x^3 - 10x^2 - 2x + 12 = 0$. **7**

OR

If $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots\ldots\ldots + a_nx^n$ is polynomial with integer co-efficient and $\frac{p}{q}$ is a rational number in lowest terms. If $\frac{p}{q}$ is a root of the equation $f(x) = 0$, then prove that $p \mid a_0$ and $q \mid a_n$.

4. (a) An ideal I in a commutative ring R with unity is maximal ideal iff the quotient ring R/I is a field. **7**

OR

Give an example of a ring in which some prime ideal is not a maximal ideal.

(b) Prove that the ideal $I = \langle x^3 - x - 1 \rangle$ is a maximal ideal in $Z_3[x]$. **7**

OR

Prove that a ring R can be embedded in a ring R' with unity.

5. Answer in short : (any **seven**) **14**

(i) Define division ring.

(ii) Is the ring $(\mathbb{Z}_7, +_7, \times_7)$ an integral domain ?

(iii) List all unit elements in a ring $(\mathbb{Z}, +, \cdot)$.

(iv) Is $(\mathbb{Z}, +, \cdot)$ an ideal of $(\mathbb{Q}, +, \cdot)$?

(v) Define the kernel of homomorphism.

(vi) Define associate polynomials.

(vii) Is the polynomial $f(x) = 9x^2 + 16$ reducible in $\mathbb{Q}(x)$? Also check its reducibility in $\mathbb{C}[x]$.

(viii) Is the ideal $\langle 6 \rangle$ prime ideal in the ring of integers ? Justify your answer,

(ix) List all zeroes of polynomial $f(x) = x^2 - 1$ in $\mathbb{Z}_{15}$.