Seat No. : _____

AE-108

April -2018

B.Sc., Sem.-VI

CC-310 : Mathematics
(Graph Theory)

Time : 3 Hours] **[Max. Marks : 70**

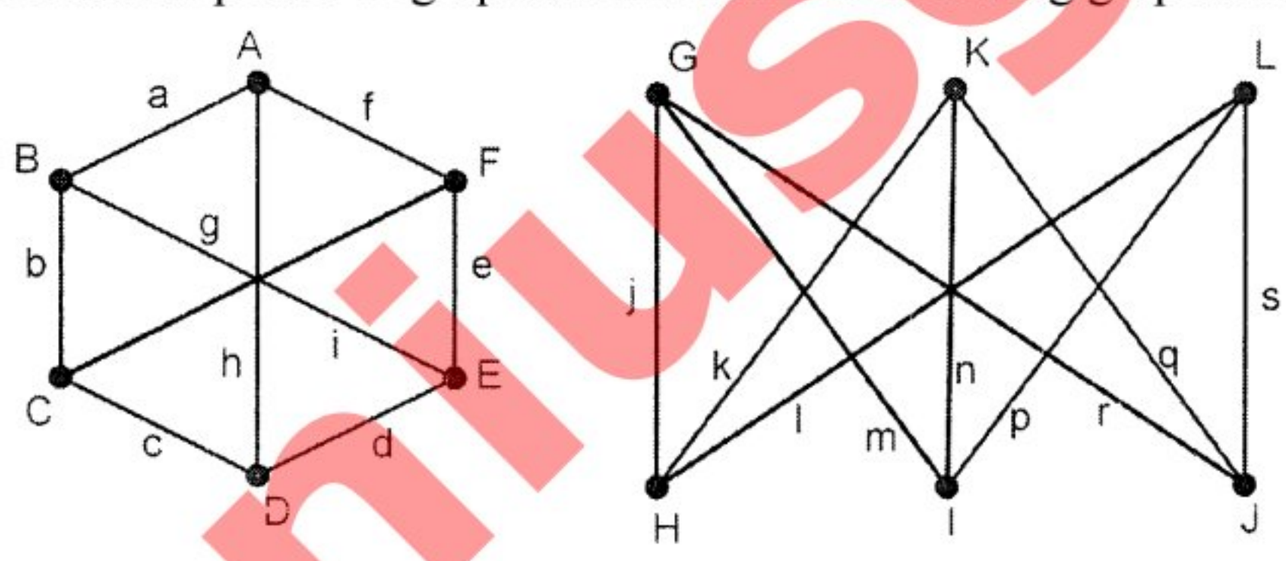
Instructions : (1) **All** questions are compulsory.
(2) Figures to the right indicate full marks of the question/sub-question.
(3) Notations used in this question paper carry their usual meaning.

1. (a) If G is any graph with e edges and n vertices $v_1, v_2, v_3, \ldots, v_n$ then prove that

$$\sum_{i=1}^{n} d(v_i) = 2e.$$ **7**

OR

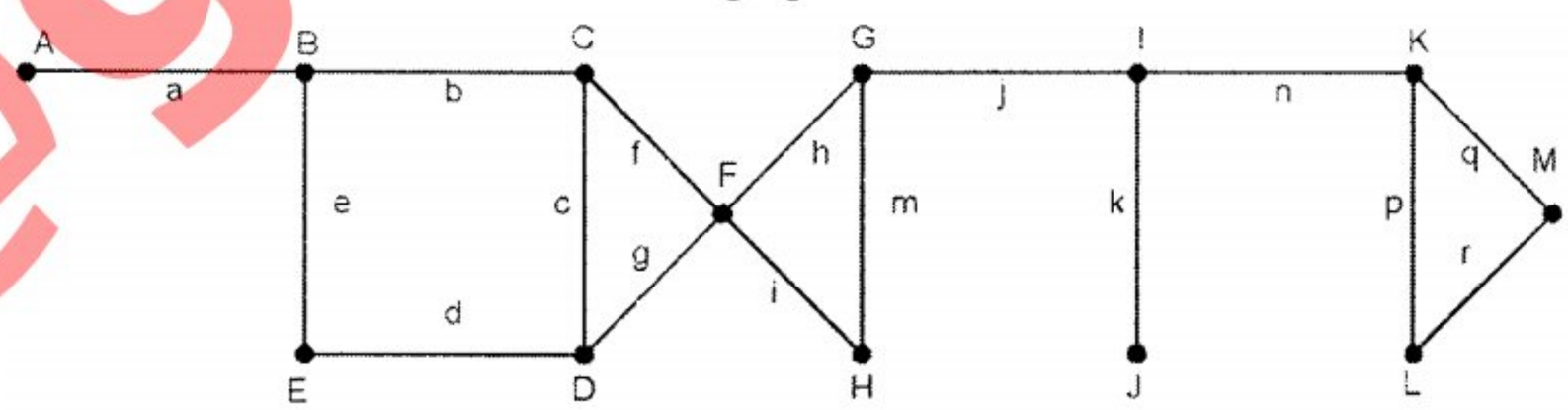
Define isomorphism of graphs. Show that the following graphs are isomorphic :



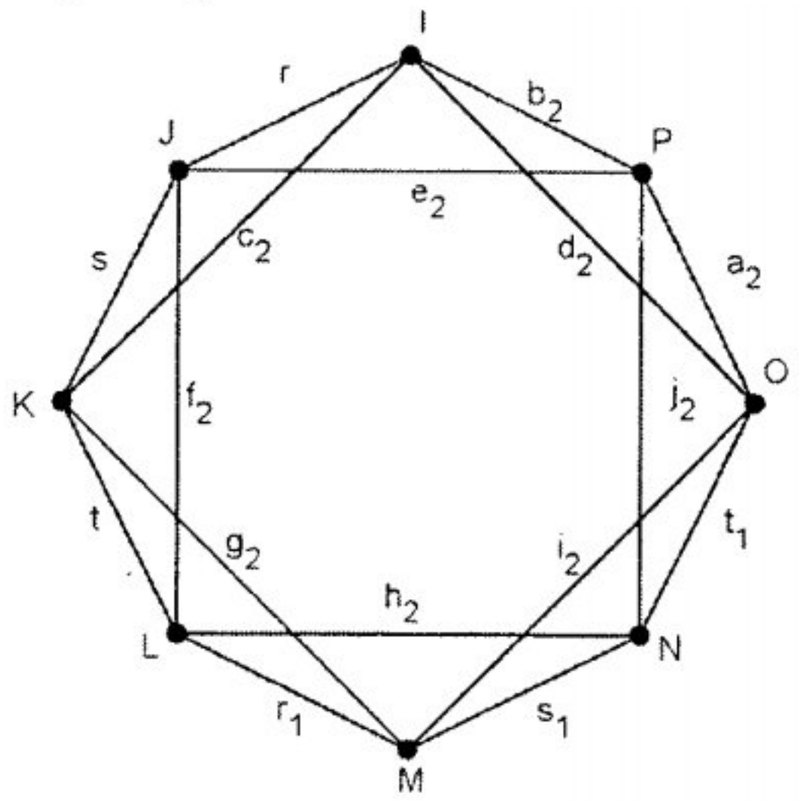
(b) Given any two vertices $u, v \in V(G)$, prove that every $u - v$ walk contains a $u - v$ path. **7**

OR

Find radius and diameter of the graph G :



2. (a) Write down the adjacency and incidence matrices of the following graph : **7**



OR

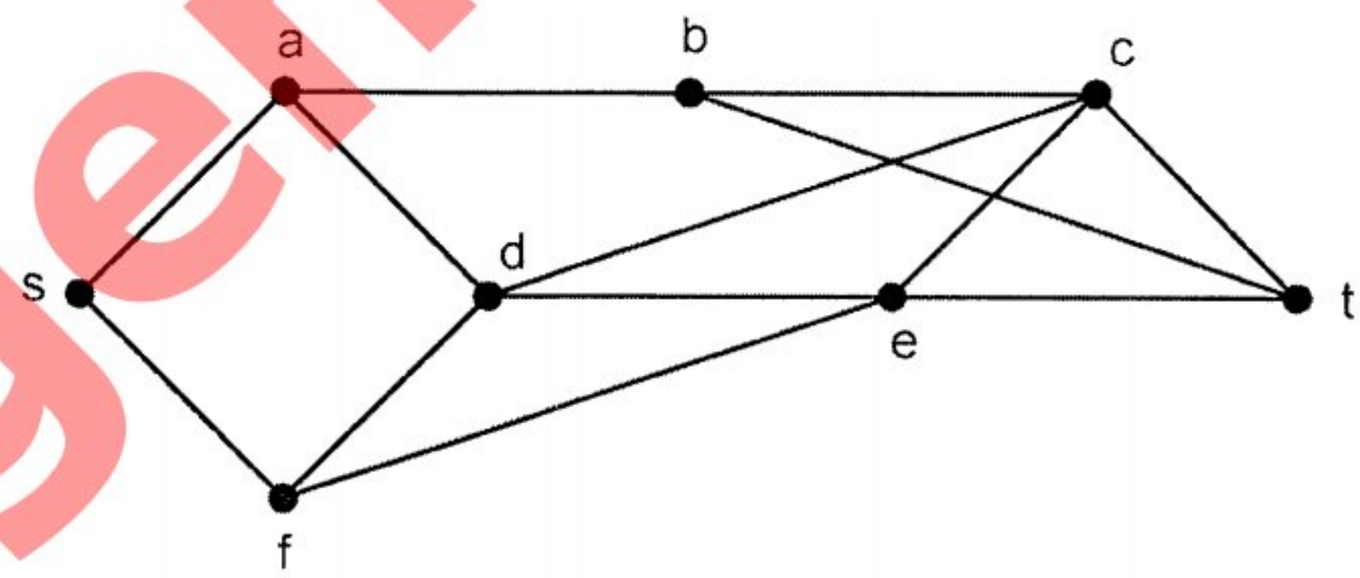
Prove that an edge 'e' of a graph G is a bridge if and only if 'e' is not part of any cycle in graph G.

(b) If G is an acyclic graph with n vertices and k connected components, then prove that it has $n - k$ edges. **7**

OR

Let G be a graph with n vertices. If G is acyclic graph with $n - 1$ edges, then prove that G is a tree.

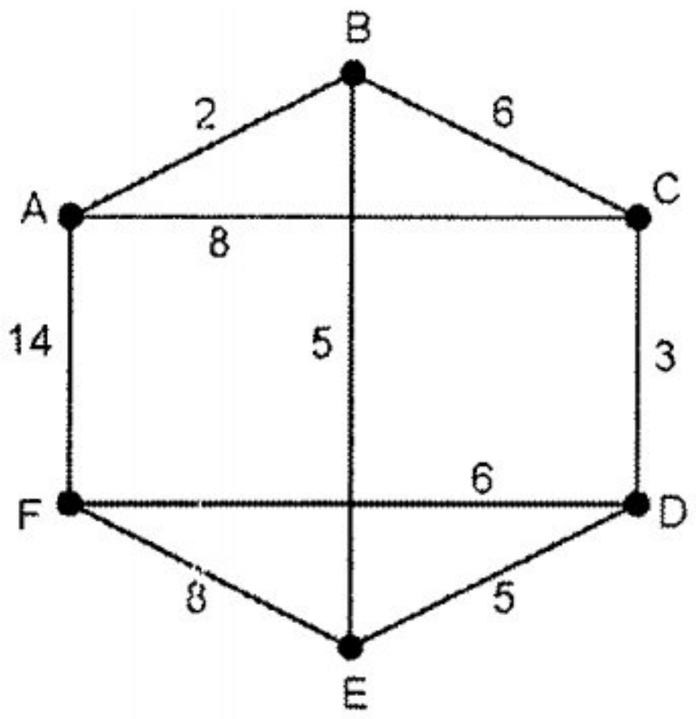
3. (a) Use Back-tracking algorithm to find a shortest path from a vertex s to a vertex t : **7**



OR

If G is a connected graph, then prove that it has a spanning tree.

(b) Apply the Dijkstra's algorithm on the following connected weighted graph to find the length of shortest paths from the vertex A to each of the other vertices of following graph : 7



OR

Let G be a graph with n vertices $n \geq 2$, then prove that G has atleast two vertices which are not cut vertices.

4. (a) Write a short note on Konigsberg seven bridges problem. 7

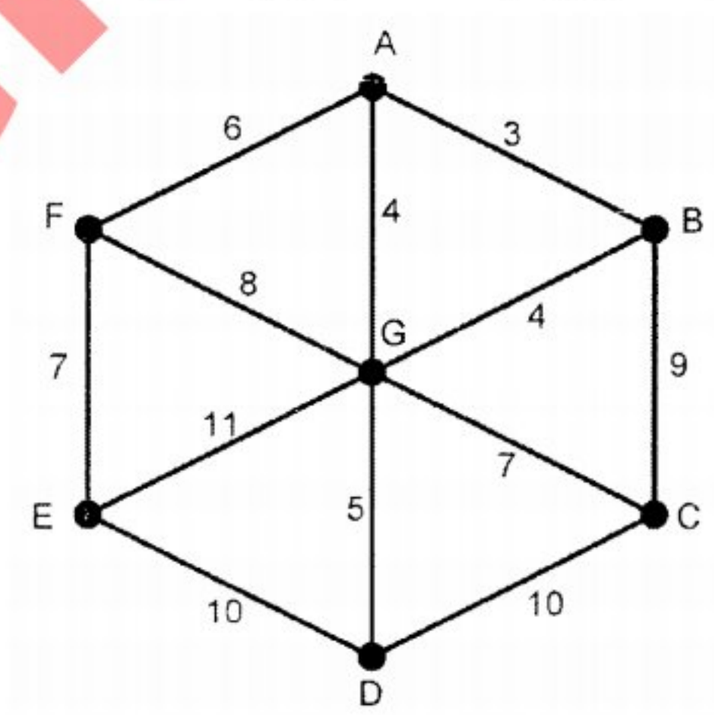
OR

If G is a graph in which the degree of every vertex is atleast two, then prove that G contains a cycle.

(b) Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian. 7

OR

Apply Prim's algorithm to find the minimal spanning tree on the graph :



5. Answer the following questions in short (any **seven**) : **14**

(1) Define cycle and give an example.

(2) Define k-regular graph and give an example of 3-regular graph.

(3) Define trail with an example.

(4) Define the distance between two vertices in a connected graph.

(5) Define Hamiltonian graph.

(6) Define *n*-connected graph.

(7) If connected graph G has 17 edges what is the maximum possible number of vertices in G ?

(8) Is the graph G with adjacency matrix $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ connected ? Why ?

(9) If connected graph G has 17 edges what is the maximum possible number of vertices in G ?
