

Seat No. : \_\_\_\_\_

**MO-128**

**March-2019**

**B.Sc., Sem.-VI**

**CC-309 : Mathematics**

**Time : 2:30 Hours]**

**[Max. Marks : 70**

- Instructions :**
- (1) All questions are compulsory.
  - (2) Right hand side figure indicates marks of that question.

1. (A) (i) Let  $X$  be a metric space. Prove that a subset  $G$  of  $X$  is open if and only if it is a union of open spheres. 7

- (ii) Prove that in any metric space  $X$ , each open sphere is an open set. 7

**OR**

- (i) Define closed set. Let  $X$  be a metric space. A subset  $F$  of  $X$  is closed if and only if its complement  $F'$  is open.

- (ii) Let  $X$  be a complete metric space and let  $Y$  be a subspace of  $X$ . Prove that  $Y$  is complete if and only if it is closed.

- (B) Attempt any **two** short questions : 4

- (1) Is the real function  $|x|$  defined on real line  $R$  is metric ? Justify.

- (2) Define metric space.

- (3) Define interior of  $A$ . Give any two basic properties of  $\text{Int}(A)$ .

2. (A) (i) Prove that closed subset of a compact sets are compact. 7

- (ii) Prove that a compact subset of a metric space are closed. 7

**OR**

- (i) A subset  $E$  of a real line  $R^1$  is connected if and only if it has following property : "If  $x \in E, y \in E$  and  $x < z < y$  then  $z \in E$ ".

- (ii) A mapping  $f$  of a metric space  $X$  into a metric space  $Y$  is continuous on  $X$  if and only if  $f^{-1}(V)$  is open in  $X$  for every open set  $V$  in  $Y$ .

- (B) Attempt any **two** short questions : 4

- (1) Define compact metric space.

- (2) Define complete metric space.

- (3) Define bounded mapping.



3. (A) (i) State and prove Weierstrass M-test. Show that  $f_n(x) = n^2 x^n(1-x)$ ;  $x \in [0, 1]$  does not converge uniformly to a function which is continuous on  $[0, 1]$ . 7

(ii) Let  $f_n$  satisfy

- (1)  $f_n \in D[a, b]$
- (2)  $(f_n(x_0))$  converges for  $x_0 \in D[a, b]$
- (3)  $f_n$  converges uniformly on  $[a, b]$  then prove that  $f_n$  converges uniformly on  $[a, b]$  to a function  $f$ . 7

**OR**

- (i) Let  $(f_n)$  be a sequence of continuous function on  $E \subset \mathbb{C}$  converges uniformly to  $f$  on  $E$  then prove that  $f$  is continuous on  $E$ .
- (ii) Prove that there exists a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  which is continuous everywhere but differentiable nowhere.

(B) Attempt any **two** short questions : 3

- (1) Is  $f_n(x) = \frac{1}{1+nx}$  ( $x \geq 0$ ) point wise convergent? justify.
- (2) If the series  $\sum a_k$  converges absolutely then prove that the series  $\sum a_k \cos(kx)$  is uniformly convergent on  $\mathbb{R}$ .
- (3) Define Uniform convergence.

4. (A) (i) Let  $f(x) = \sum a_n x^n$  be a power series with radius of convergence 1. If the series converges at 1 then prove that  $\lim_{x \rightarrow 1^-} f(x) = f(1)$  7

(ii) State and prove Weierstrass Approximation theorem. 7

**OR**

- (i) For every  $x \in \mathbb{R}$ , and  $n > 0$ , prove that

$$\sum_{k=0}^n (nx - k)^2 \binom{n}{k} x^k (1-x)^{n-k} = nx(1-x) \leq \frac{n}{4}$$

- (ii) Show that for  $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$  for  $-1 \leq x \leq 1$ . Hence deduce that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

(B) Attempt any **two** short questions : 3

- (1) Show that  $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$
- (2) State Binomial series for  $\alpha \in \mathbb{R}$  and  $|x| < 1$ .
- (3) Define Taylor's series.