Seat No. : ______________

SJ-128

September-2020

B.Sc., Sem.-VI

CC-308 : Mathematics
(Analysis-II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) All Questions in **Section I** carry equal marks.
(2) Attempt any **THREE** questions in **Section I**.
(3) Question IX in **Section II** is **COMPULSORY**.

Section – I

Attempt any **Three** questions :

1. (A) Let f be integrable on [a, b] and $a < c < b$, then prove that f is integrable on [a, c] and [c, b] and $\int_a^b f = \int_a^c f + \int_c^b f$. 7

(B) Let $f(x)=2x^2/3$ on [0, 1] for n $\in$ $NP_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \frac{4}{n} \ldots \frac{n-1}{n}, 1\right\}$, then find $\lim\limits_{n\to\infty} U[f;P_n]$ *and* $\lim\limits_{n\to\infty} L[f;P_n]$. 7

2. (A) State and prove Second Mean Value Theorem of Integral Calculus. 7

(B) Prove that $\frac{1}{3\sqrt{2}} \leq \int_0^1 \frac{x^2}{\sqrt{1+x}} dx \leq \frac{1}{3}$ 7

3. (A) Prove that the series $\sum \frac{1}{n!} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!} + \ldots$ converges to the value e, which is an irrational number ? 7

(B) Prove that if p > 1, the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges and if p $\leq$ 1, the series diverges. 7

4. (A) State and prove Cauchy's condemsation test. 7

(B) Test for convergence :

(1) $\sum_{n=1}^{\infty} \frac{n^{5/2}}{n^2 + 3n + 5}$ (2) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{-n^2}$ 7

5. (A) State and Prove Merten's Theorem. **7**

(B) Find the set of convergence (interval of convergence) and radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n(x-1)^n}{(n+1)5^n}$. **7**

6. (A) If $\sum a_n$ is absolutely convergent, then prove that any rearrangement of $\sum a_n$ has the same sum. **7**

(B) For the following, determine whether the series converges absolutely, converges conditionally, or diverges :

(1) $\sum \frac{(-1)^n n}{(n^2+1)}$ (2) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^{3/2}}$ **7**

7. (A) Obtain Maclaurin series expansion of $\sin x$ for $-\infty < x < \infty$. **7**

(B) Write Taylor's formula with Cauchy form of remainder for $f(x) = (1-x)^{1/2}$ about $a = 0$ and $-1 < x < 1$. **7**

8. (A) Let f be a real valued function on [a, a + h] and $f^{n+1}(x)$ is continuous on [a, a + h]. Then Prove that,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + .. + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_{n+1}(x) \text{ for}$$

$x \in [a, a+h]$

Where $R_{n+1}(x) = \frac{1}{n!}\int_a^x (x-t)^n f^{(n+1)}(t)dt$. **7**

(B) Let $(1-x)y' + 1 = 0$ with initial conditions $y(0)=1$. Find a power series solution for this equation in power of x. **7**

Section – II

9. Attempt any **Four** short questions : **8**

(1) Give an example of a sequence which is bounded and divergent series.

(2) If $f(x) = 3\cos x - 2e^x$, find the primitive F of f.

(3) Find limit superior and limit inferior of the sequence $S_n = \{1, 1/2, 1/3, 1/4, ..\}$.

(4) Write Maclaurin series expansion of $\log(1+x)$ for $-1 < x < 1$.

(5) Test for convergence : $\int_0^{\infty} \frac{dx}{1+x^2} dx$.

(6) Find the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.